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Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed six manuscript pages and three figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Irreversible Thermal-Diffusional Coupling in Local Equilibrium

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Nomenclature

D_j = constant resembling the diffusional phenomenological coupling coefficient,

$$\sum_{i=1}^{p}\left(\delta_{ij}-\frac{n_i}{n}\right)\langle E(\boldsymbol{r},t)\rangle_i u/n\Delta a$$

$E(v)$ = total energy contained in a local volume of the mixed gas,

$$\sum_{i=1}^{p} E(v_i)$$

$E(v_i)$ = total energy of the ith specie in a local volume of the gas

$\{\hat{e}_i\}$ = set of linearly independent basis

$f_i(\boldsymbol{v}_i,\boldsymbol{r},t)$ = local equilibrium distribution function for the ith specie, $n_i(\boldsymbol{r},t)[m_i\beta(\boldsymbol{r},t)/2\pi]^{3/2} \times\exp\{-[m_i\beta(\boldsymbol{r},t)/2][\boldsymbol{v}_i-\boldsymbol{u}(\boldsymbol{r},t)]^2\}$

$\boldsymbol{J}(t)$ = total heat-flux density of the mixed gas across $\Delta\boldsymbol{a}$,

$$\sum_{i=1}^{p}\boldsymbol{J}_i(t)$$

$\boldsymbol{J}_i(t)$ = heat-flux density of the ith specie, $\frac{1}{2}\rho_i v_i^2\boldsymbol{v}_i$

k_B = Boltzmann constant

m_i = mass of a particle in the ith specie

N_i = total number of particles of the ith specie in δV

$n(\boldsymbol{r},t)$ = total number density,

$$\sum_{i=1}^{p} n_i(\boldsymbol{r},t)$$

$n_i(\boldsymbol{r},t)$ = number density of the ith specie

$\hat{n}$ = unit vector

p = total number of species available in the mixed gas

$\boldsymbol{r}$ = position vector of an arbitrary point in the mixed gas

T = temperature

t = time

$\boldsymbol{u}(\boldsymbol{r},t)$ = average velocity or the stream velocity vector of the gas

$\boldsymbol{v}_i$ = velocity of a particle in the ith specie

α_i = dimensionless number, $m_i(\langle v_i^2\rangle_i+u^2-2\boldsymbol{u}\cdot\langle\boldsymbol{v}_i\rangle_i)/3k_BT$

β = $1/k_BT$

β_i^* = Eucken number of the ith specie, $3(1-\alpha_i)$

γ = physical constant,

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\left[\left(\delta_{ij}-\frac{n_i}{n}\right)-\delta_{ij}(1-\alpha_i)\right]\times(\langle E(\boldsymbol{r},t)\rangle_i n_j/n\Delta a)$$

Δ = variable's nonequilibrium instantaneous departure from its equilibrium value

$\Delta\boldsymbol{a}$ = total area,

$$\int_{\text{open surface}}\mathrm{d}\boldsymbol{a}$$

δ = fluctuation in a variable

δ_{ij} = Kronecker delta

δV = infinitesimal volume element

∂_t = derivative with respect to time, $\partial/\partial t$

ϵ = perturbation

θ = time as an integration variable

λ_i = thermal conductivity of the ith specie, $\beta_i^*\langle E(\boldsymbol{r},t)\rangle_i u/2T\Delta a$

$\bar{\lambda}$ = constant resembling the thermal phenomenological coupling coefficient,

$$\sum_{i=1}^{p} n_i\lambda_i/n$$

ρ_i = mass density of the ith species, $m_iN_i/\delta V$

τ = relaxation time

Υ_i = mulitiplicative factor, $\frac{1}{2}(3\{1-[m_i(\langle v_i^2\rangle_i+u^2-2\boldsymbol{u}\cdot\langle\boldsymbol{v}_i\rangle_i)/3k_BT]\})=\frac{1}{2}3(1-\alpha_i)=\frac{1}{2}\beta_i^*$

$\langle\,\rangle$ = nonequilibrium ensemble average

$\langle\,\rangle_e$ = equilibrium ensemble average

$\langle(\cdots)\rangle$ = average over the constituents of the gas,

$$\sum_{i=1}^{p}\int \mathrm{d}^3v_i(\cdots)f_i(\boldsymbol{v}_i,\boldsymbol{r},t)/n(\boldsymbol{r},t)$$

$\langle(\cdots)\rangle_i$ = average over the ith specie,

$$\int \mathrm{d}^3v_i(\cdots)f_i(\boldsymbol{v}_i,\boldsymbol{r},t)/n_i(\boldsymbol{r},t)$$

Introduction

COUPLING phenomena of irreversible processes are said to constitute the subject of nonequilibrium thermodynamics.[1] The equations written for these coupled irreversible processes are all phenomenological. They are also referred to as the mutual interaction of irreversible processes.[2] The derivation of some of these relations are presented by solving the Boltzmann transport equation using approximation techniques.[3] In this Note using the method of energy dynamics, which was used before for monoatomic and diatomics gases,[4,5] the derivation of thermal-diffusional coupling is presented for a mixture of monoatomic gases that is in a local equilibrium state. The method of energy dynamics does not involve

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any approximations as the one needed for the Boltzmann transport equation. Moreover, one gets a heat-flux equation that will indicate heat flow with finite speed. Heat flow caused by particle diffusion is called the Dufour effect, and it was demonstrated qualitatively for a mixture of gases.[2] An analytical derivation of heat flow in a mixture of monoatomic gases caused by the existence of particle gradients, which can be interpreted as the term responsible for the Dufour effect, is presented here. The Dufour-effect term obtained by the method of successive approximation of the Boltzmann transport equation[3] is a sum of diffusion constants multiplied by relative particle diffusion velocities. Taking the same expression to represent the Dufour effect in local equilibrium state would indicate no heat flow as a result of particle diffusion, which is because the diffusion velocities are all zeros in local equilibrium, thus indicating the absence of thermal-diffusional coupling in local equilibrium. Here by applying the method of energy dynamics[4] to a gas with mixed species at local equilibrium, one can still get a contribution to heat flow caused by particle diffusion.

Analytical Presentation of the Dufour Effect

To be able to get a heat-flux equation with finite speed for the case of a mixture of gases, one must recover the equation that relates the relaxation of nonequilibrium quantities to the regression of spontaneous fluctuations. Such a relation, which is sometimes referred to as the Onsager hypothesis, is a result of the classical formula of linear response theory.[6] When applying the linear response theory to a mixture of monoatomic gases, one must consider the application of the theory to quantities that are defined for the total gas. Therefore, because $\langle \boldsymbol{J}_i(t)\rangle_e = 0;\ \forall i = 1, \ldots, p$, applying the linear response theory to the flux of total kinetic energy in a mixture of monoatomic gases would imply[4]

$$
\begin{aligned}
\langle \boldsymbol{J}(t)\rangle = \langle \Delta \boldsymbol{J}(t)\rangle &= \frac{\beta\epsilon}{3}\sum_{l=1}^{3}\langle \delta\boldsymbol{J}(0)\cdot\delta\boldsymbol{J}(t)\rangle_e\,\hat{e}_l \\
&= \frac{\beta\epsilon}{3}\sum_{l=1}^{3}\sum_{i,j=1}^{p}\left(\frac{\rho_i\rho_j}{4}\right)\left\langle v_i^2(0)\boldsymbol{v}_i(0)\cdot v_j^2(t)\boldsymbol{v}_j(t)\right\rangle_e\hat{e}_l \\
&= \frac{\beta\epsilon}{3}\sum_{l=1}^{3}\sum_{i=1}^{p}\left(\frac{\rho_i}{2}\right)^2\left\langle v_i^2(0)\boldsymbol{v}_i(0)\cdot v_i^2(t)\boldsymbol{v}_i(t)\right\rangle_e\hat{e}_l \\
&\le \frac{\beta\epsilon}{3}\sum_{i,l,k}\left(\frac{\rho_i}{2}\right)^2\langle v_{ki}(0)v_{ki}(t)\rangle_e^3\,\hat{e}_l
\end{aligned}
\tag{1}
$$

where

$$
\left\langle v_i^2(0)\boldsymbol{v}_i(0)\cdot v_j^2(t)\boldsymbol{v}_j(t)\right\rangle_e \equiv \delta_{ij}\left\langle v_i^2(0)\boldsymbol{v}_i(0)\cdot v_j^2(t)\boldsymbol{v}_j(t)\right\rangle_e
$$

which is a consequence of having a mixture of Maxwellian molecules and where correlations are assumed isotropic. Recalling the heat-flux equation that leads to heat conduction with finite signal time, which was[4]

$$
\tau\partial_t\langle \boldsymbol{J}(\boldsymbol{r}, t)\rangle + \langle \boldsymbol{J}(\boldsymbol{r}, t, t)\rangle = -\lambda\nabla T - \gamma\hat{n}(\nabla\cdot\boldsymbol{u}) \tag{2}
$$

and observing the relation provided by Eq. (1), one can immediately recover the left-hand side of Eq. (2) by applying the method of energy dynamics to the energy balance equation for mixed gases, as was done before for monoatomic gases.[4] The difference in the case of mixed gases should appear on the right-hand side of Eq. (2). To apply the method of energy dynamics to a mixture of monoatomic gases, one must start with the conservation of energy equation

$$
\int_0^t \mathrm{d}\theta\boldsymbol{J}(\boldsymbol{v}, \theta)\cdot\Delta\boldsymbol{a} = -E(v) \tag{3}
$$

where one chooses an appropriate volume such that Eq. (3) is true.[4] In the case of a mixture of gases, Eq. (3) is written as

$$
\int_0^t \mathrm{d}\theta\sum_{i=1}^{p}\boldsymbol{J}_i(\boldsymbol{v}_i, \theta)\cdot\Delta\boldsymbol{a} = -\sum_{i=1}^{p}E(v_i) \tag{4}
$$

which can also be written as

$$
\int_0^t \mathrm{d}\theta\left[\frac{\sum_{i=1}^{p}\int \mathrm{d}^3v_i\boldsymbol{J}_i(\boldsymbol{v}_i, \theta)f_i(\boldsymbol{v}_i, \boldsymbol{r}, t)}{\sum_{i=1}^{p}n_i(\boldsymbol{r}, t)}\right]\cdot\Delta\boldsymbol{a} = -\sum_{i=1}^{p}\frac{\int \mathrm{d}^3v_iE(v_i)f_i(\boldsymbol{v}_i, \boldsymbol{r}, t)}{\sum_{i=1}^{p}n_i(\boldsymbol{r}, t)} \tag{5}
$$

or

$$
\int_0^t \mathrm{d}\theta\langle \boldsymbol{J}(\boldsymbol{r}, t, \theta)\rangle\cdot\Delta\boldsymbol{a} = -\sum_{i=1}^{p}\frac{n_i(\boldsymbol{r}, t)\langle E(\boldsymbol{r}, t)\rangle_i}{n(\boldsymbol{r}, t)} \tag{6}
$$

The evolution of the system in Eq. (6) is governed by the equation

$$
\left[\int_0^t \mathrm{d}\theta\partial_t\langle \boldsymbol{J}(\boldsymbol{r}, t, \theta)\rangle + \langle \boldsymbol{J}(\boldsymbol{r}, t, t)\rangle\right]\cdot\Delta\boldsymbol{a} = -\partial_t\sum_{i=1}^{p}\frac{n_i(\boldsymbol{r}, t)\langle E(\boldsymbol{r}, t)\rangle_i}{n(\boldsymbol{r}, t)} \tag{7}
$$

where the right-hand side of Eq. (7) is given by

$$
-\partial_t\sum_{i=1}^{p}\frac{n_i(\boldsymbol{r}, t)\langle E(\boldsymbol{r}, t)\rangle_i}{n(\boldsymbol{r}, t)} = \sum_{i=1}^{p}\frac{\int \mathrm{d}^3v_iE(v_i)\partial_tf_i(\boldsymbol{v}_i, \boldsymbol{r}, t)}{n(\boldsymbol{r}, t)} - \langle E(\boldsymbol{r}, t)\rangle\frac{\partial_tn(\boldsymbol{r}, t)}{n(\boldsymbol{r}, t)} \tag{8}
$$

with

$$
\langle E(\boldsymbol{r}, t)\rangle = \sum_{i=1}^{p}\frac{n_i(\boldsymbol{r}, t)\langle E(\boldsymbol{r}, t)\rangle_i}{n(\boldsymbol{r}, t)}
$$

$$
\langle E(\boldsymbol{r}, t)\rangle_i = \frac{\int \mathrm{d}^3v_iE(v_i)f_i(\boldsymbol{v}_i, \boldsymbol{r}, t)}{n_i(\boldsymbol{r}, t)}
$$

To evaluate the first term in the right-hand side of Eq. (8), one must find an expression for the time derivative of $f_i(\boldsymbol{v}_i, \boldsymbol{r}, t)$, and by choosing a local Maxwellian one gets

$$
\begin{aligned}
\partial_tf_i(\boldsymbol{v}_i, \boldsymbol{r}, t) &= \partial_tn_i(\boldsymbol{r}, t)\left[\frac{m_i\beta(\boldsymbol{r}, t)}{2\pi}\right]^{\frac{3}{2}} \\
&\quad\times\exp\left\{-\frac{m_i\beta(\boldsymbol{r}, t)}{2}[\boldsymbol{v}_i - \boldsymbol{u}(\boldsymbol{r}, t)]^2\right\} \\
&\quad+ n_i(\boldsymbol{r}, t)\frac{3}{2}\left[\frac{m_i\beta(\boldsymbol{r}, t)}{2\pi}\right]^{\frac{3}{2}-1}\frac{m_i}{2\pi}\partial_t\beta(\boldsymbol{r}, t) \\
&\quad\times\exp\left\{-\frac{m_i\beta(\boldsymbol{r}, t)}{2}[\boldsymbol{v}_i - \boldsymbol{u}(\boldsymbol{r}, t)]^2\right\} \\
&\quad- n_i(\boldsymbol{r}, t)\left[\frac{m_i\beta(\boldsymbol{r}, t)}{2\pi}\right]^{\frac{3}{2}}\frac{m_i}{2}\partial_t\beta(\boldsymbol{r}, t)[\boldsymbol{v}_i - \boldsymbol{u}(\boldsymbol{r}, t)]^2 \\
&\quad\times\exp\left\{-\frac{m_i\beta(\boldsymbol{r}, t)}{2}[\boldsymbol{v}_i - \boldsymbol{u}(\boldsymbol{r}, t)]^2\right\} \\
&\quad+ n_i(\boldsymbol{r}, t)\left[\frac{m_i\beta(\boldsymbol{r}, t)}{2\pi}\right]^{\frac{3}{2}}\beta(\boldsymbol{r}, t)m_i\partial_t\boldsymbol{u}(\boldsymbol{r}, t)[\boldsymbol{v}_i - \boldsymbol{u}(\boldsymbol{r}, t)] \\
&\quad\times\exp\left\{-\frac{m_i\beta(\boldsymbol{r}, t)}{2}[\boldsymbol{v}_i - \boldsymbol{u}(\boldsymbol{r}, t)]^2\right\}
\end{aligned}
\tag{9}
$$

By substituting Eq. (9) into Eq. (8), it is then possible to evaluate the first term in the right-hand side of Eq. (8):

$$\sum_{i=1}^{p}\frac{\int d^3v_i E(v_i)\partial_t f_i(\boldsymbol{v}_i,\boldsymbol{r},t)}{n(\boldsymbol{r},t)} = \frac{1}{n(\boldsymbol{r},t)}\sum_{i=1}^{p}\Bigg\{\langle E(\boldsymbol{r},t)\rangle_i\partial_t n_i(\boldsymbol{r},t) + n_i(\boldsymbol{r},t)\bigg[\langle E(\boldsymbol{r},t)\rangle_i\frac{3}{2}\beta^{-1}\partial_t\beta - \left\langle E(v_i)(\boldsymbol{v}_i-\boldsymbol{u})^2\right\rangle_i\frac{m_i}{2}\partial_t\beta + \langle E(v_i)(\boldsymbol{v}_i-\boldsymbol{u})\rangle_i m_i\cdot\beta\partial_t\boldsymbol{u}\bigg]\Bigg\} \tag{10}$$

Introducing the approximations[4]

$$\langle E(v_i)\boldsymbol{v}_i\rangle_i = \langle E(\boldsymbol{r},t)\rangle_i\langle\boldsymbol{v}_i\rangle_i$$

$$\left\langle E(v_i)v_i^2\right\rangle_i = \langle E(\boldsymbol{r},t)\rangle_i\left\langle v_i^2\right\rangle_i$$

along with[5]

$$\partial_t\beta = (\beta/T)\boldsymbol{u}\cdot\nabla T + (2/3)\beta\nabla\cdot\boldsymbol{u}$$

allows one to write Eq. (10) as

$$\sum_{i=1}^{p}\frac{\int d^3v_i E(v_i)\partial_t f_i(\boldsymbol{v}_i,\boldsymbol{r},t)}{n(\boldsymbol{r},t)} = \frac{1}{n}\sum_i \Upsilon_i n_i\langle E(\boldsymbol{r},t)\rangle_i\frac{\boldsymbol{u}}{T}\cdot\nabla T + \frac{1}{n}\sum_i \Upsilon_i n_i\langle E(\boldsymbol{r},t)\rangle_i\frac{2}{3}\nabla\cdot\boldsymbol{u} + \frac{1}{n}\sum_i\langle E(\boldsymbol{r},t)\rangle_i\partial_t n_i + \frac{1}{n}\sum_i n_i m_i(\langle\boldsymbol{v}_i\rangle_i-\boldsymbol{u})\langle E(\boldsymbol{r},t)\rangle_i\beta\cdot\partial_t\boldsymbol{u} \tag{11}$$

However, because $f_i(\boldsymbol{v}_i,\boldsymbol{r},t)$ is a local Maxwellian, the computations are facilitated by the equality $\langle\boldsymbol{v}_i\rangle_i = \boldsymbol{u}$; $\forall i = 1,\ldots,p$, which will give $\delta v_i^2 = \langle v_i^2\rangle_i - \langle v_i\rangle_i^2$. Hence Eq. (11) can be written as

$$\sum_{i=1}^{p}\frac{\int d^3v_i E(v_i)\partial_t f_i(\boldsymbol{v}_i,\boldsymbol{r},t)}{n(\boldsymbol{r},t)} = \frac{1}{2}\left[\frac{1}{n}\sum_i\beta_i^* n_i\langle E(\boldsymbol{r},t)\rangle_i\frac{\boldsymbol{u}}{T}\right]\cdot\nabla T + \frac{1}{n}\sum_i(1-\alpha_i)n_i\langle E(\boldsymbol{r},t)\rangle_i\nabla\cdot\boldsymbol{u} + \frac{1}{n}\sum_i\langle E(\boldsymbol{r},t)\rangle_i\partial_t n_i \tag{12}$$

Equation (8) together with Eq. (12) will give

$$-\partial_t\sum_{i=1}^{p}\frac{n_i(\boldsymbol{r},t)\langle E(\boldsymbol{r},t)\rangle_i}{n(\boldsymbol{r},t)} = -\frac{1}{2}\left[\frac{1}{n}\sum_i\beta_i^* n_i\langle E(\boldsymbol{r},t)\rangle_i\frac{\boldsymbol{u}}{T}\right]\cdot\nabla T - \frac{1}{n}\sum_i(1-\alpha_i)n_i\langle E(\boldsymbol{r},t)\rangle_i\nabla\cdot\boldsymbol{u} + \sum_i\sum_j\left(\frac{n_i-n\delta_{ij}}{n}\right)\frac{\langle E(\boldsymbol{r},t)\rangle_i}{n}\partial_t n_j \tag{13}$$

and because the continuity of species equation[3] is given by

$$\partial_t n_j = -\nabla\cdot(n_j\langle\boldsymbol{v}_j\rangle_j) = -n_j\nabla\cdot\boldsymbol{u} - \boldsymbol{u}\cdot\nabla n_j \tag{14}$$

one may eliminate the time derivative of n_j in the right-hand side of Eq. (13) in favor of the gradient of n_j:

$$\sum_{i,j}\left(\frac{n_i-n\delta_{ij}}{n}\right)\frac{\langle E(\boldsymbol{r},t)\rangle_i}{n}\partial_t n_j = -\sum_{i,j}\left(\frac{n_i-n\delta_{ij}}{n}\right)\frac{\langle E(\boldsymbol{r},t)\rangle_i}{n}n_j\nabla\cdot\boldsymbol{u} - \sum_{i,j}\left(\frac{n_i-n\delta_{ij}}{n}\right)\frac{\langle E(\boldsymbol{r},t)\rangle_i}{n}\boldsymbol{u}\cdot\nabla n_j \tag{15}$$

Combining Eqs. (7), (13), and (15), one obtains the result that

$$\left[\int_0^t d\theta\,\partial_t\langle\boldsymbol{J}(\boldsymbol{r},t,\theta)\rangle + \langle\boldsymbol{J}(\boldsymbol{r},t,t)\rangle\right]\cdot\Delta\boldsymbol{a} = -\frac{1}{n}\sum_i n_i\left[\frac{1}{2}\beta_i^*\langle E(\boldsymbol{r},t)\rangle_i\frac{\boldsymbol{u}}{T}\right]\cdot\nabla T + \left[\sum_{i,j}\left(\frac{n\delta_{ij}-n_i}{n}\right)\frac{\langle E(\boldsymbol{r},t)\rangle_i}{n}n_j - \frac{1}{n}\sum_i(1-\alpha_i)n_i\langle E(\boldsymbol{r},t)\rangle_i\right]\nabla\cdot\boldsymbol{u} + \sum_{i,j}\left(\frac{n\delta_{ij}-n_i}{n}\right)\frac{\langle E(\boldsymbol{r},t)\rangle_i\boldsymbol{u}}{n}\cdot\nabla n_j \tag{16}$$

Taking into account the fact that $\Delta\boldsymbol{a} = \Delta a\hat{n}$ and $\boldsymbol{u} = u\hat{n}$ and making use of the Ergodic theory,[4] Eq. (16) can be written as

$$\tau\partial_t\langle\boldsymbol{J}(\boldsymbol{r},t)\rangle + \langle\boldsymbol{J}(\boldsymbol{r},t,t)\rangle = -\bar{\lambda}\nabla T + \gamma\hat{n}\nabla\cdot\boldsymbol{u} + \sum_j D_j\nabla n_j \tag{17}$$

which in the incompressible limit reduces to

$$\tau\partial_t\langle\boldsymbol{J}(\boldsymbol{r},t)\rangle + \langle\boldsymbol{J}(\boldsymbol{r},t,t)\rangle = -\bar{\lambda}\nabla T + \sum_j D_j\nabla n_j \tag{18}$$

The last term in Eq. (18) exhibits the Dufour effect. In Eq. (18) heat is flowing because of two kinds of thermodynamic affinities or forces: one involves the grad T, and the other is as a result of grad n_js and that demonstrates the phenomenon of irreversible thermal-diffusional coupling. For a single component gas one recovers the equality $\langle E(\boldsymbol{r},t)\rangle_i = \langle E(\boldsymbol{r},t)\rangle$; $\forall i = 1,\ldots,p$, which will reduce Eq. (17) to Eq. (2), which is a result that one expects. If the relaxation time τ, which was found to be to the order of picoseconds,[4] is small enough such that the first term on the left-hand side of Eq. (18) can be ignored, then the resulting equation will precisely represent the phenomenological relation of interaction between heat conduction and diffusion.[2]

Summary

The macroscopic phenomenological relation of coupled irreversible thermal-diffusional processes can be analytically derived from the hypothesis of statistical mechanics, which assumes discretized microscopic structure for bulk matter.

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Enhancement of Natural Convection by Eccentricity of Power Cable Inside Underground Conduit

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Nomenclature

D_h	=	hydraulic diameter
e	=	eccentricity
Gr	=	Grashof number, Ra/Pr
g	=	gravitational acceleration
k	=	thermal conductivity
l	=	radial distance between the inner and outer cylinders
Nu	=	Nusselt number, $h^* D_h^* / k_{\text{ref}}^*$
Pr	=	Prandtl number, υ^* / α^*
q	=	heat flux
Ra^o	=	modified Rayleigh number, $g^* \beta^* D_h^{*4} q_w^* / \upsilon^* \alpha^* k^*$
T	=	temperature
T_b	=	bulk temperature
$T_{s,\max}$	=	maximum cable surface temperature
V	=	velocity
(x, y, z)	=	Cartesian coordinates, $(x^*, y^*, z^*)/ D_h^*$
α	=	thermal diffusivity
β	=	thermal expansion coefficient
θ	=	nondimensionalized temperature, $(T^* - T_{\text{ref}}^*)/ q_w^* D_h^* / k^*$
υ	=	kinematic viscosity
ϕ	=	azimuthal angle

Subscripts

b	=	bulk
i	=	inner cylinder
l	=	local
o	=	outer cylinder
ref	=	reference state (at atmospheric pressure and room temperature)
w	=	wall

Superscripts

$-$	=	averaged quantity
$*$	=	dimensional quantity

Introduction

PLACING the power cable underground is a current engineering tendency due to the limited available space, particularly in city areas and industrial zones. The power cable (inner cylinder) is placed inside a concrete conduit (outer cylinder) buried underground. The configuration of this layout is an annulus between two horizontal, highly eccentric cylinders as schematically shown in Fig. 1. Heat is generated due to the electrical resistance of the power cable, and the heat dissipation process in the annulus relies on the natural convection heat transfer from both open ends of the conduit, which penetrate onto the manhole surfaces. As described in our previous study,[1] the geometric configuration of this eccentric annulus between two horizontal cylinders and its associated thermal boundary conditions lead to a three-dimensional, noncavity-type problem, which was seldom reported on in published work. An up-to-date review on the natural convection heat transfer in the annulus between two horizontal cylinders for two-/three-dimensional and concentric/eccentric configurations may be found in our previous work[1] and is not repeated here.

It was found from our previous work[1] that the highest temperature of the power cable is always located at the contacting point of the cable and the concrete conduit. The cause can be apparently understood from the azimuthal distributions of the local Rayleigh number, which is defined by

$$Ra_l(\phi, z) = \frac{g^* \beta_{\text{ref}}^* \left[T_i^*(\phi, z) - T_o^*(\phi, z)\right] l^{*3}}{\upsilon_{\text{ref}}^* \alpha_{\text{ref}}^*} \tag{1}$$

where l^* is the radial distance between the inner (cable) and outer (concrete conduit) surfaces, with the pole located at the center of the inner cylinder, at a given azimuthal angle ϕ and longitudinal position z. For the ordinary configuration of the cable inside an underground conduit, the cable (inner cylinder) lies on the bottom of the concrete conduit (outer cylinder). Clearly, the l^* value approaches zero as ϕ moves to the contacting point. As a result, the Rayleigh number $Ra_l(\propto l^{*3})$ in the neighborhood of the contacting point drops steeply to very small values. By definition, the Rayleigh number is equal to the Grashof number times the Prandtl number, and the Grashof number provides a measure of the ratio of the buoyancy force to the viscous force acting on the fluid. At small Rayleigh number Ra_l, the local heat transfer is mainly through the heat conduction process, and this leads to a poor heat dissipation rate.

It is known[2–6] that natural convection heat transfer rate in the annulus between two horizontal cylinders can be enhanced by

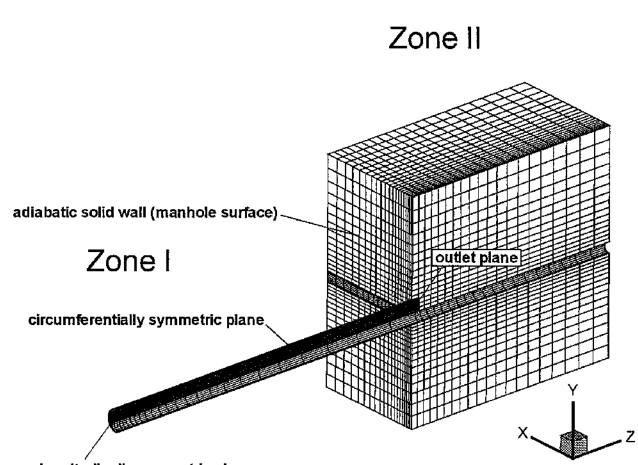

Fig. 1 Computational domain and zonal grid distribution. Origin (0, 0, 0) is located at the top point (for $e = 0.5$) or the bottom point (for the other cases) of the inner cylinder on the longitudinally symmetric plane.

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